MATH 1650 TRANSFORMATIONS WORKSHEET

NAME: ______________________________

1. Let $f(x) = x^2 - x - 6$.

   Match the functional formula on the left with the expression on the right.

   (a) $f(x+1) = (x+1)^2 - (x+1) - 6$

   (b) $f(x) + 1 = x^2 - x - 5$

   (c) $f(x-2) = (x-2)^2 - (x-2) - 6$

   (d) $f(x) - 2 = x^2 - x - 8$

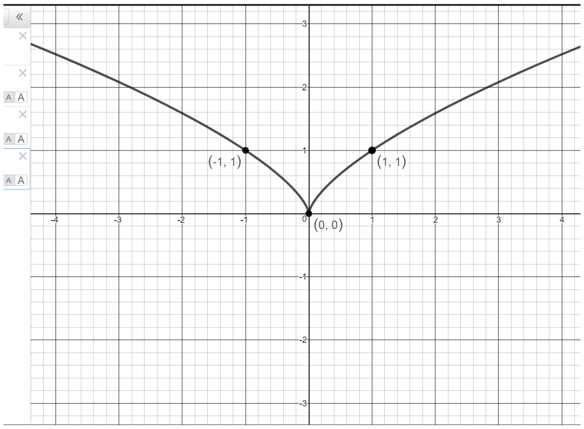
   (e) $f(3x) = 9x^2 - 3x - 6$

   (f) $3f(x) = 3x^2 - 3x - 18$

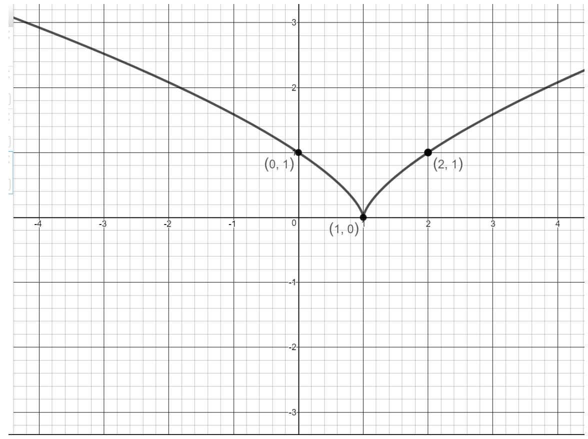
   (g) $f(-x) = x^2 + x - 6$

   (h) $-f(x) = -x^2 + x + 6$

2. Consider each pair of graphs below, and answer the following questions:

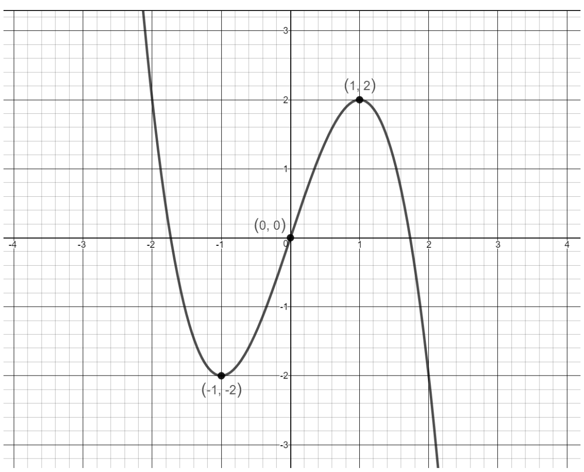


$y = x^{\frac{2}{3}}$

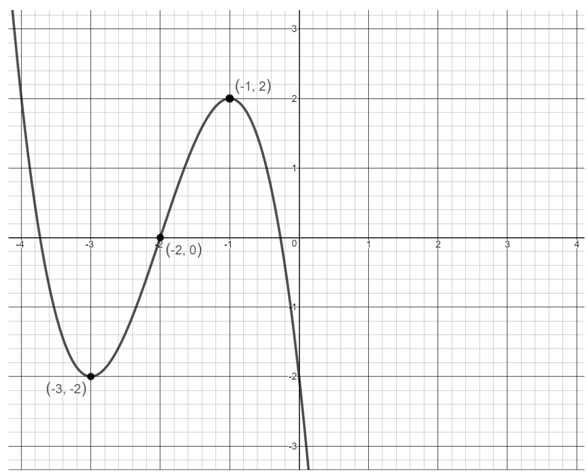


$y = (x-1)^{\frac{2}{3}}$

- How are these two graphs related geometrically?
  The graph of $y = (x-1)^{\frac{2}{3}}$ is the graph of $y = x^{\frac{2}{3}}$ shifted right one unit.
- How are these two formulas related algebraically?
  We replace x in the formula $y = x^{\frac{2}{3}}$ with $(x-1)$ to get $y = (x-1)^{\frac{2}{3}}$.



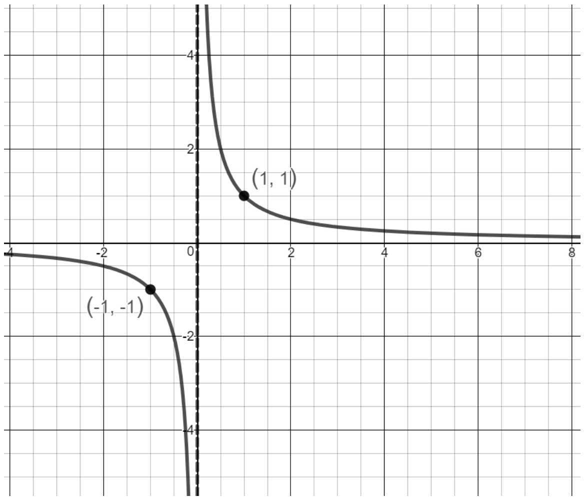
$y = 3x - x^3$



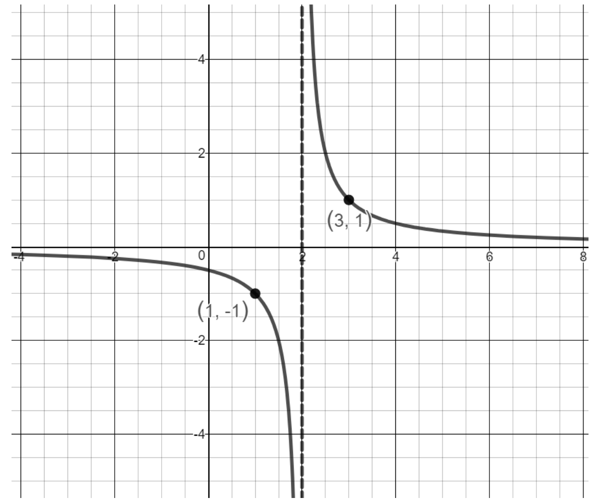
$y = 3(x+2) - (x+2)^3$

- How are these two graphs related geometrically?
  The graph of $y = 3(x+2) - (x+2)^3$ is the graph of $y = 3x - x^3$ but shifted left two units.
- How are these two formulas related algebraically?
  We replace x in the formula $y = 3x - x^3$ with $(x+2)$ to get $y = 3(x+2) - (x+2)^3$.

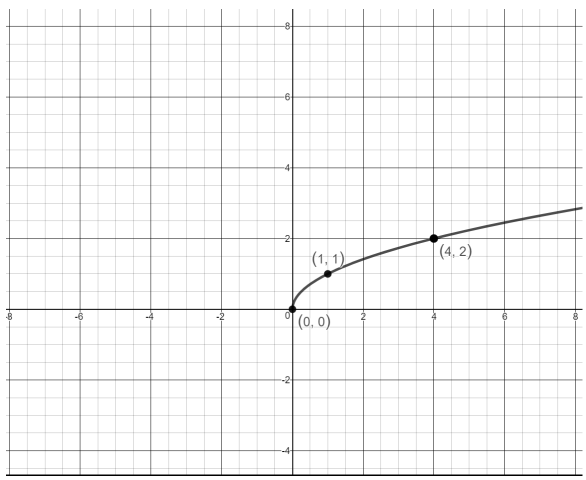
3. Given the formula for the function graphed on the left, find a formula for the function on the right.



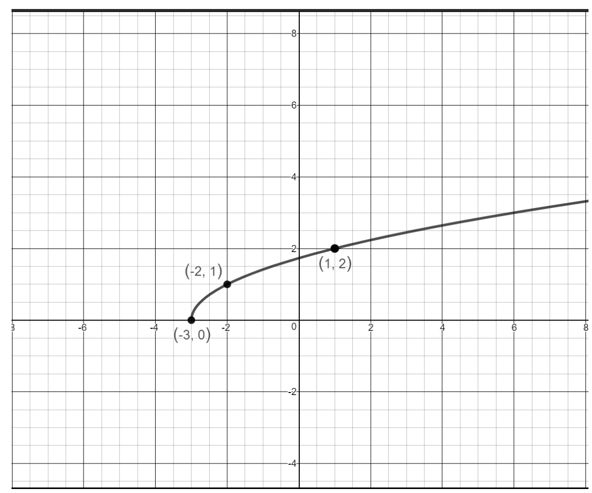
$$y = \frac{1}{x}$$



$$y = \frac{1}{x-2}$$



$$y = \sqrt{x}$$



$$y = \sqrt{x+3}$$

4. If $(3, -4)$ is on the graph of $y = f(x)$, find a point on the graph of:

(a) $y = f(x-2)$: $(5, -4)$

(b) $y = f(x+1)$ $(2, -4)$

5. If (c, d) is on the graph of $y = f(x)$, find a point on the graph of:

(a) $y = f(x-2)$: $(c+2, d)$

(b) $y = f(x+1)$: $(c-1, d)$

6. If (c, d) is on the graph of $y = f(x)$ and h is a positive number, find a point on the graph of:

(a) $y = f(x-h)$: $(c+h, d)$

(b) $y = f(x+h)$: $(c-h, d)$

7. If $h > 0$, what is the geometric difference between the graphs of:

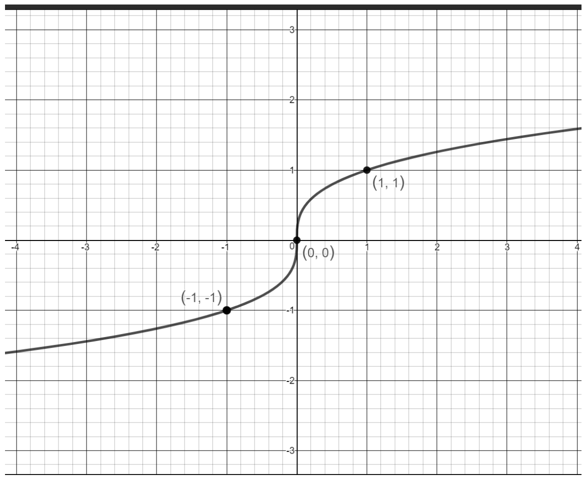
- $y = f(x)$ and $y = f(x-h)$?

Move all points right h units.

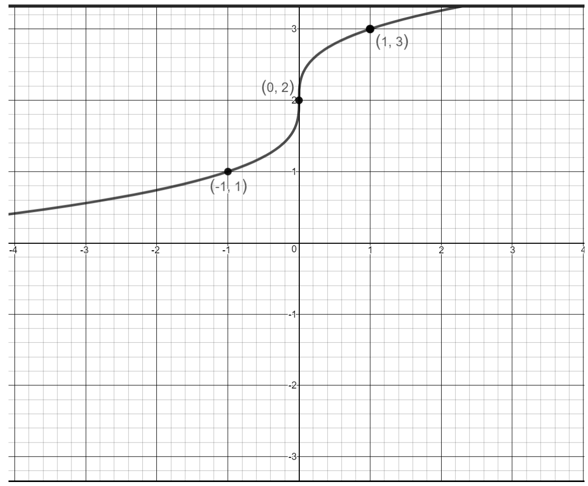
- $y = f(x)$ and $y = f(x+h)$?

Move all points left h units.

8. Consider each pair of graphs below, and answer the following questions:

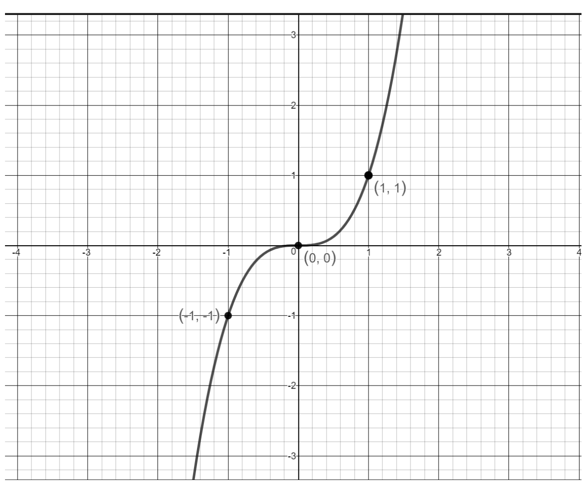


$y = \sqrt[3]{x}$

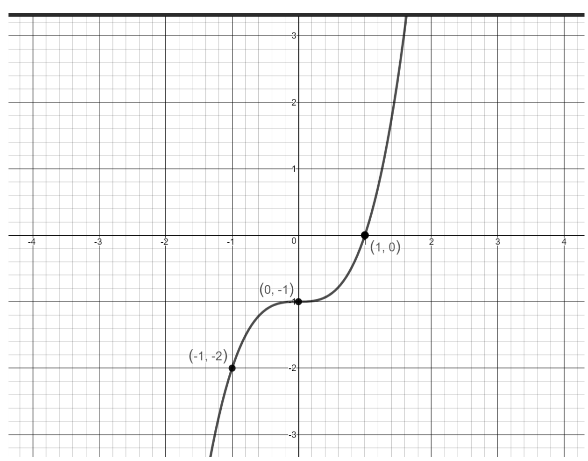


$y = \sqrt[3]{x} + 2$

- How are these two graphs related geometrically?
  The graph of $y = \sqrt[3]{x} + 2$ is the graph of $y = \sqrt[3]{x}$ but shifted up two units.
- How are these two formulas related algebraically?
  We add 2 to the formula $\sqrt[3]{x}$ to get $y = \sqrt[3]{x} + 2$.



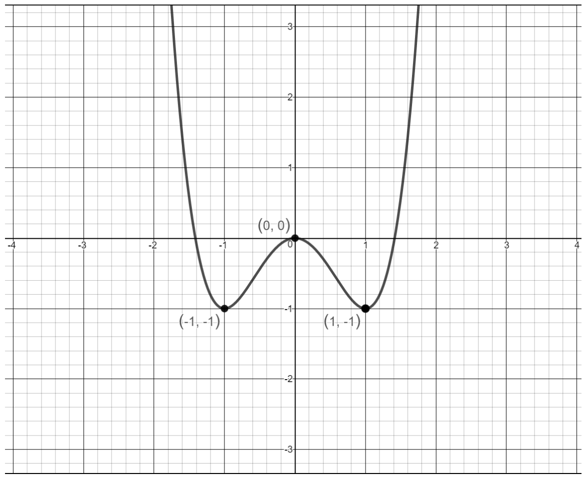
$y = x^3$



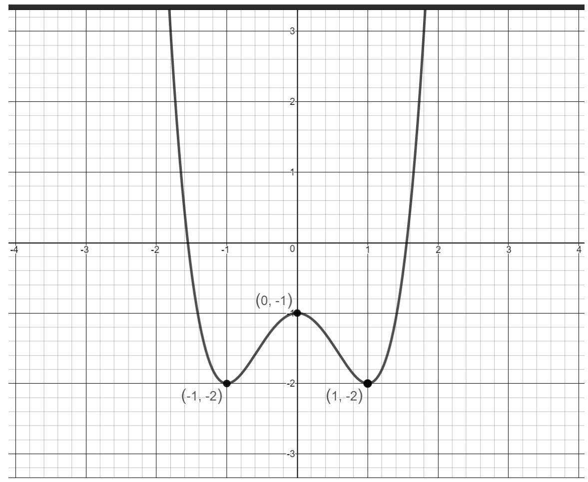
$y = x^3 - 1$

- How are these two graphs related geometrically?
  The graph of $y = x^3 - 1$ is the graph of $y = x^3$ but shifted down 1 unit.
- How are these two formulas related algebraically?
  We subtract 1 from the formula x^3 to get $y = x^3 - 1$.

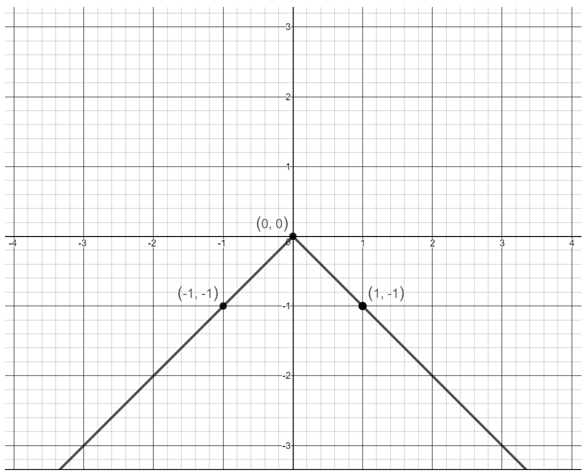
9. Given the formula for the function graphed on the left, find a formula for the function on the right.



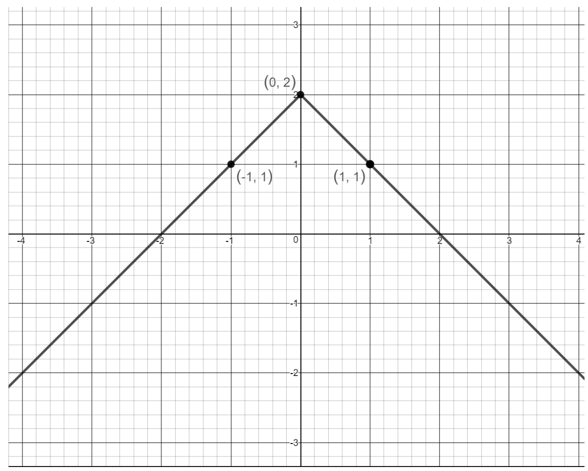
$y = x^4 - 2x^2$



$y = x^4 - 2x^2 - 1$



$y = -|x|$



$y = -|x| + 2$

10. If $(3, -4)$ is on the graph of $y = f(x)$, find a point on the graph of:

(a) $y = f(x) - 2$: $(3, -6)$

(b) $y = f(x) + 1$: $(3, -3)$

11. If (c, d) is on the graph of $y = f(x)$, find a point on the graph of:

(a) $y = f(x) - 2$: $(c, d - 2)$

(b) $y = f(x) + 1$: $(c, d + 1)$

12. If (c, d) is on the graph of $y = f(x)$ and k is a positive number, find a point on the graph of:

(a) $y = f(x) - k$: $(c, d - k)$

(b) $y = f(x) + k$: $(c, d + k)$

13. If $k > 0$, what is the geometric difference between the graphs of:

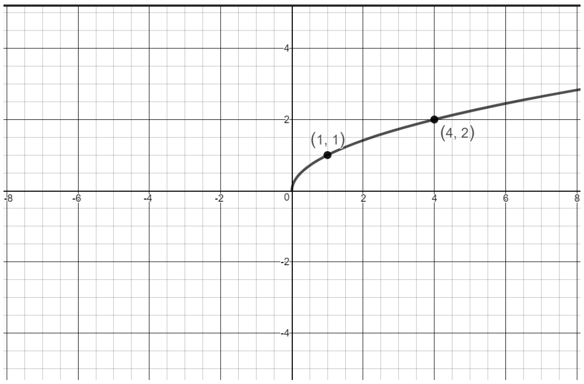
- $y = f(x)$ and $y = f(x) - k$?

Move all the points down k units.

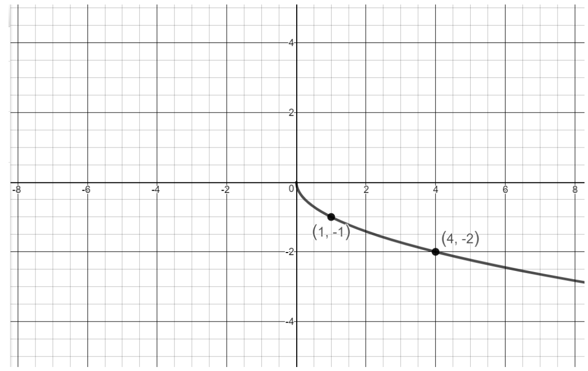
- $y = f(x)$ and $y = f(x) + k$?

Move all the points up k units.

14. Consider each pair of graphs below, and answer the following questions:

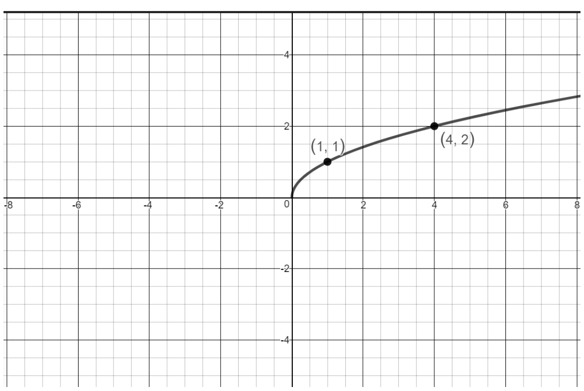


$y = \sqrt{x}$

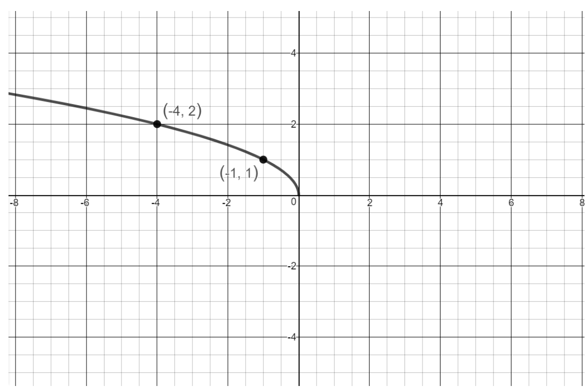


$y = -\sqrt{x}$

- How are these two graphs related geometrically?
  They are reflections about the x-axis.
- How are these two formulas related algebraically?
  We multiply $\sqrt{x}$ by -1 to obtain the formula $y = -\sqrt{x}$.



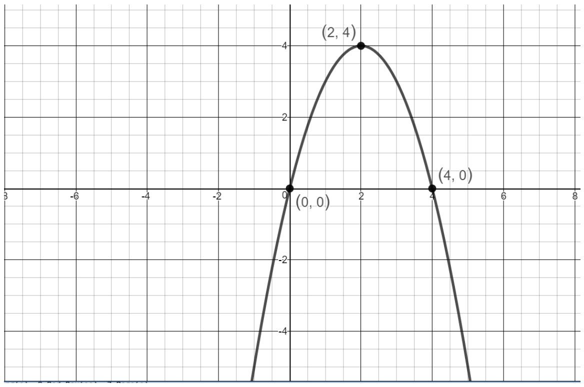
$y = \sqrt{x}$

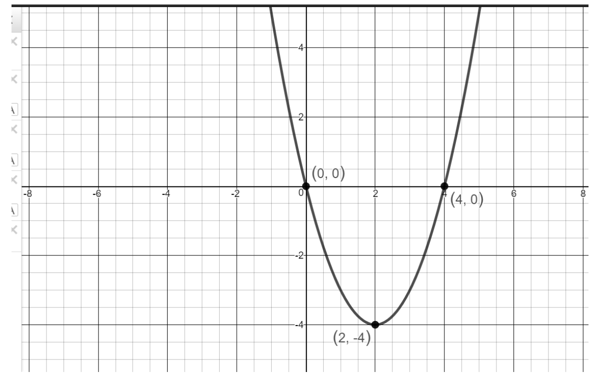


$y = \sqrt{-x}$

- How are these two graphs related geometrically?
  They are reflections about the y-axis.
- How are these two formulas related algebraically?
  We replace x in the formula $y = \sqrt{x}$ with $-x$ to obtain the formula $y = \sqrt{-x}$.

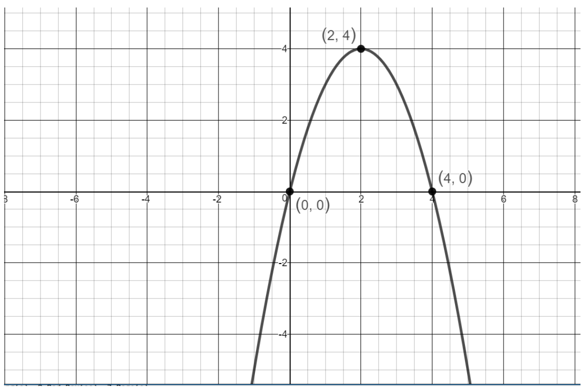
15. Given the formula for the function graphed on the left, find a formula for the function on the right.

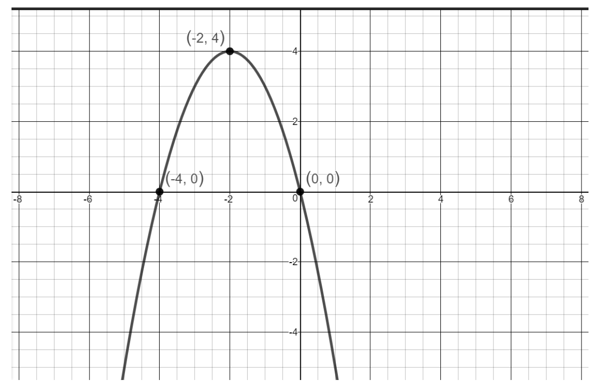




$y = 4x - x^2$

$y = -\left(4x - x^2\right) = -4x + x^2$





$y = 4x - x^2$

$y = 4(-x) - (-x)^2 = -4x - x^2$

16. If $(3, -4)$ is on the graph of $y = f(x)$, find a point on the graph of:

(a) $y = -f(x)$: $(3, 4)$

(b) $y = f(-x)$: $(-3, -4)$

17. If (c, d) is on the graph of $y = f(x)$, find a point on the graph of:

(a) $y = -f(x)$: $(c, -d)$

(b) $y = f(-x)$: $(-c, d)$

18. What is the geometric difference between the graphs of:

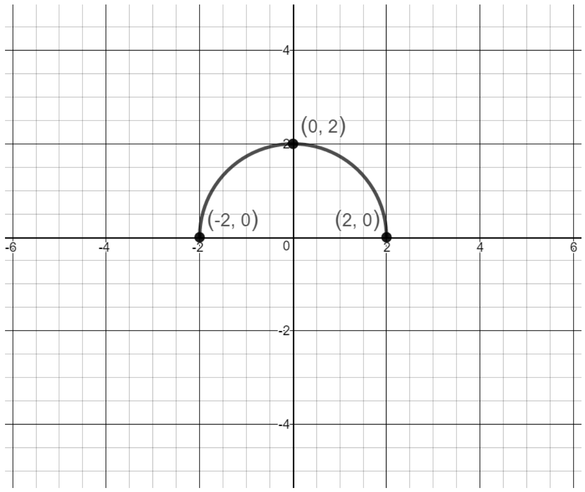
- $y = f(x)$ and $y = -f(x)$?

They are reflections across the x-axis.

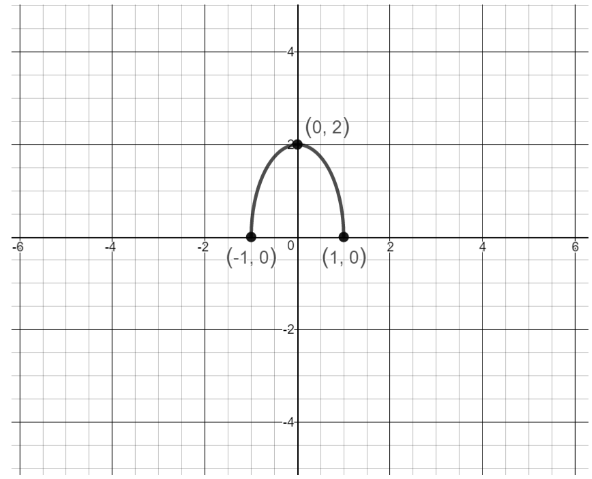
- $y = f(x)$ and $y = f(-x)$?

They are reflections across the y-axis.

19. Consider each pair of graphs below, and answer the following questions:

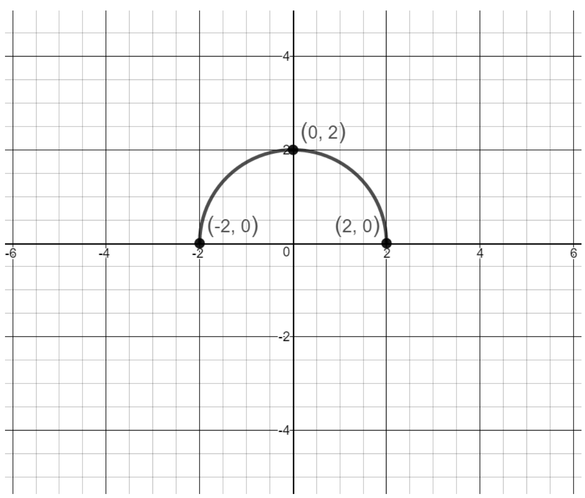


$y = \sqrt{4 - x^2}$

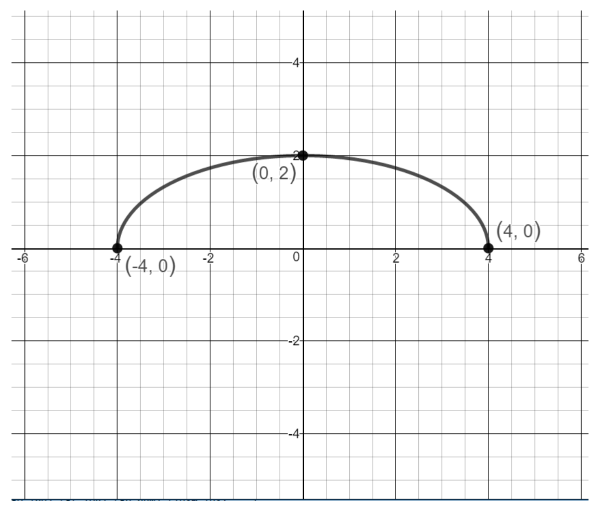


$y = \sqrt{4 - (2x)^2}$

- How are these two graphs related geometrically?
  The graph of $y = \sqrt{4 - (2x)^2}$ is two times as close to the y-axis as the graph of $y = \sqrt{4 - x^2}$.
- How are these two formulas related algebraically?
  We replace x in the formula $y = \sqrt{4 - x^2}$ with $2x$ to obtain $y = \sqrt{4 - (2x)^2}$.



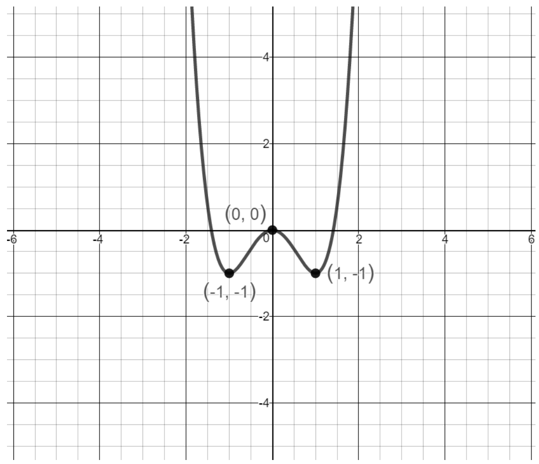
$y = \sqrt{4 - x^2}$



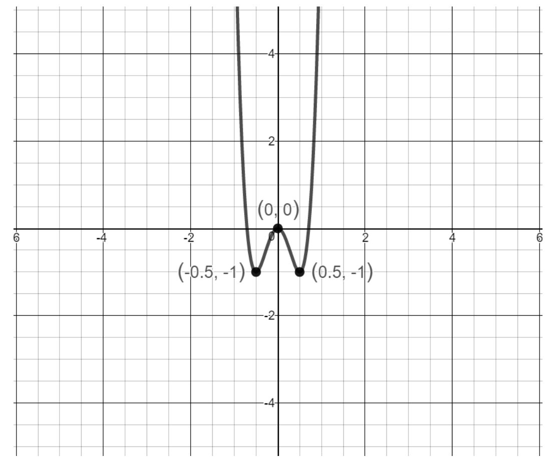
$y = \sqrt{4 - \left(\frac{x}{2}\right)^2}$

- How are these two graphs related geometrically?
  The graph of $y = \sqrt{4 - \left(\frac{x}{2}\right)^2}$ is two times as far away from the y-axis as the graph of $y = \sqrt{4 - x^2}$.
- How are these two formulas related algebraically?
  We replace x in the formula $y = \sqrt{4 - x^2}$ with $\frac{x}{2}$ to obtain $y = \sqrt{4 - \left(\frac{x}{2}\right)^2}$.

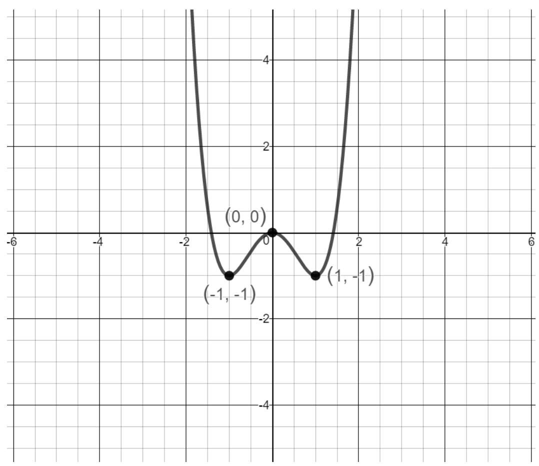
20. Given the formula for the function graphed on the left, find a formula for the function on the right.



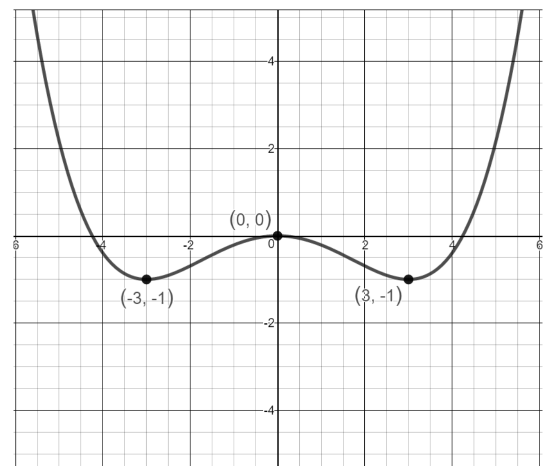
$y = x^4 - 2x^2$



$y = (2x)^4 - 2(2x)^2 = 16x^4 - 8x^2$



$y = x^4 - 2x^2$



$y = \left(\frac{x}{3}\right)^4 - 2\left(\frac{x}{3}\right)^2 = \dfrac{x^4}{81} - \dfrac{2x^2}{9}$

21. If $(3, -4)$ is on the graph of $y = f(x)$, find a point on the graph of:

(a) $y = f(5x)$: $\left(\frac{3}{5}, -4\right)$

(b) $y = f\left(\frac{x}{5}\right)$: $(15, -4)$

22. If (c, d) is on the graph of $y = f(x)$, find a point on the graph of:

(a) $y = f(5x)$: $\left(\frac{c}{5}, d\right)$

(b) $y = f\left(\frac{x}{5}\right)$: $(5c, d)$

23. If (c, d) is on the graph of $y = f(x)$ and b is a positive number, find a point on the graph of:

(a) $y = f(bx)$: $\left(\frac{c}{b}, d\right)$

(b) $y = f\left(\frac{x}{b}\right)$: (bc, d)

24. If $b > 1$, what is the geometric difference between the graphs of:

- $y = f(x)$ and $y = f(bx)$?

Divide all x-values by b.

Horizontal compression of the graph.

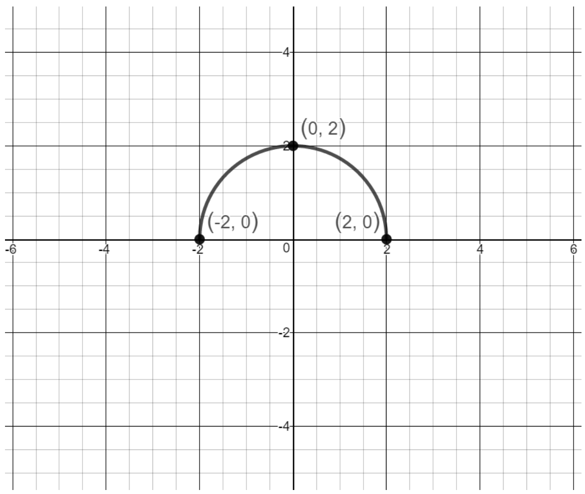
- $y = f(x)$ and $y = f\left(\frac{x}{b}\right)$?

Multiply all x-values by b.

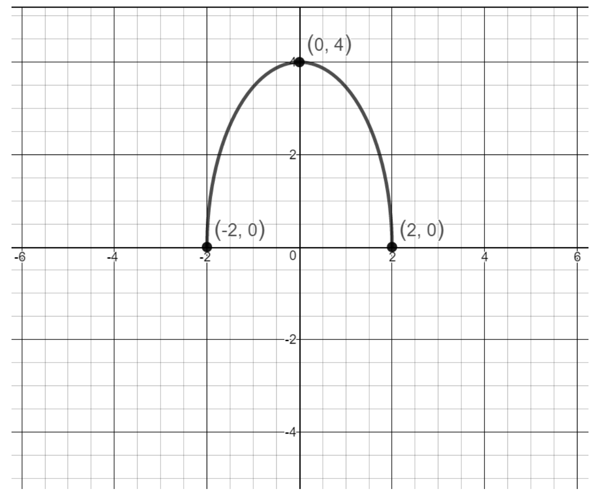
Horizontal stretch of the graph.

If $0 < b < 1$, dividing by b stretches the graph horizontally and multiplying by b compresses the graph.

25. Consider each pair of graphs below, and answer the following questions:

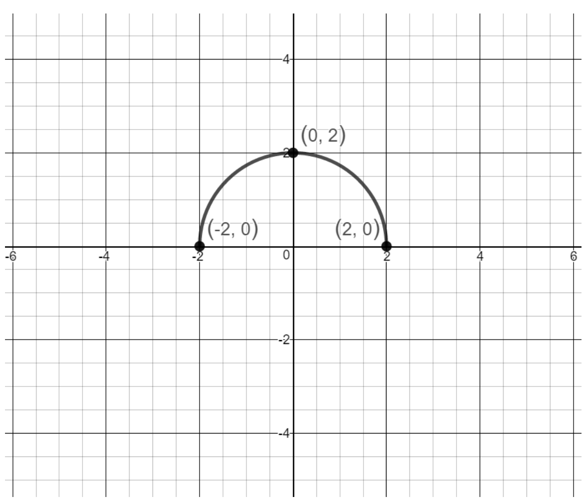


$y = \sqrt{4 - x^2}$

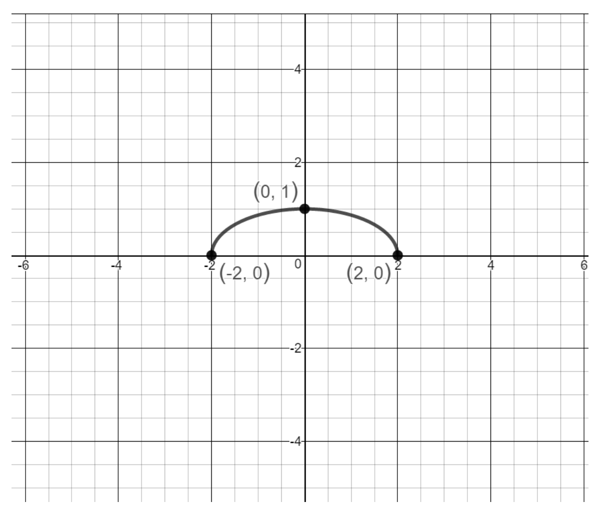


$y = 2\sqrt{4 - x^2}$

- How are these two graphs related geometrically?
  The graph of $y = 2\sqrt{4 - x^2}$ is twice as far away from the x-axis as $y = \sqrt{4 - x^2}$.
- How are these two formulas related algebraically?
  The formula $\sqrt{4 - x^2}$ is multiplied by 2 to obtain $y = 2\sqrt{4 - x^2}$.



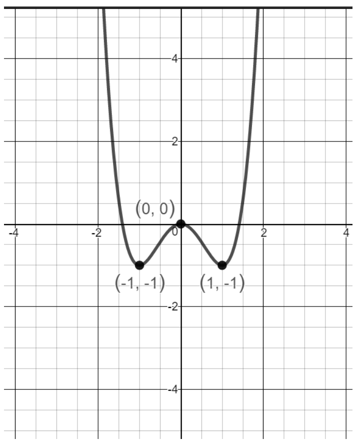
$y = \sqrt{4 - x^2}$



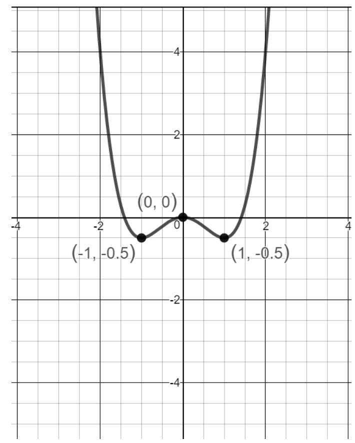
$y = \dfrac{1}{2}\sqrt{4 - x^2}$

- How are these two graphs related geometrically?
  The graph of $y = \frac{1}{2}\sqrt{4 - x^2}$ is half as far away from the x-axis as $y = \sqrt{4 - x^2}$.
- How are these two formulas related algebraically?
  The formula by $\sqrt{4 - x^2}$ is multiplied by $\frac{1}{2}$ to obtain the formula $y = \frac{1}{2}\sqrt{4 - x^2}$.

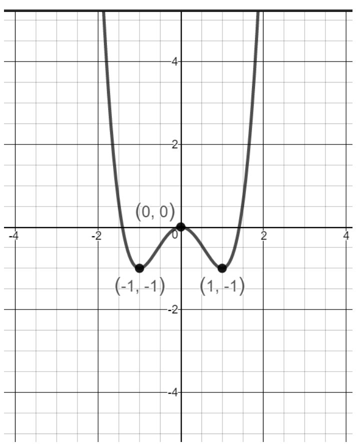
26. Given the formula for the function graphed on the left, find a formula for the function on the right.



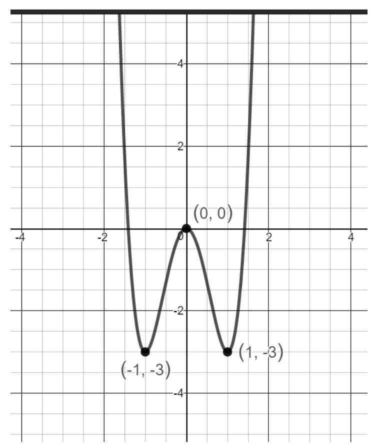
$y = x^4 - 2x^2$



$y = \frac{1}{2}\left(x^4 - 2x^2\right) = \frac{1}{2}x^4 - x^2$



$y = x^4 - 2x^2$



$y = 3\left(x^4 - 2x^2\right) = 3x^4 - 6x^2$

27. If $(3, -4)$ is on the graph of $y = f(x)$, find a point on the graph of:

(a) $y = 5f(x)$: $(3, -20)$

(b) $y = \dfrac{1}{5}f(x)$: $\left(3, -\frac{4}{5}\right)$.

28. If (c, d) is on the graph of $y = f(x)$, find a point on the graph of:

(a) $y = 5f(x)$: $(c, 5d)$

(b) $y = \frac{1}{5}f(x)$: $\left(c, \frac{d}{5}\right)$

29. If (c, d) is on the graph of $y = f(x)$ and a is a positive number, find a point on the graph of:

(a) $y = af(x)$: (c, ad)

(b) $y = \frac{1}{a}f(x)$: $\left(c, \frac{d}{a}\right)$

30. If $a > 1$, what is the geometric difference between the graphs of:

- $y = f(x)$ and $y = af(x)$?

Multiply all the y-values by a.

Stretches the graph vertically.

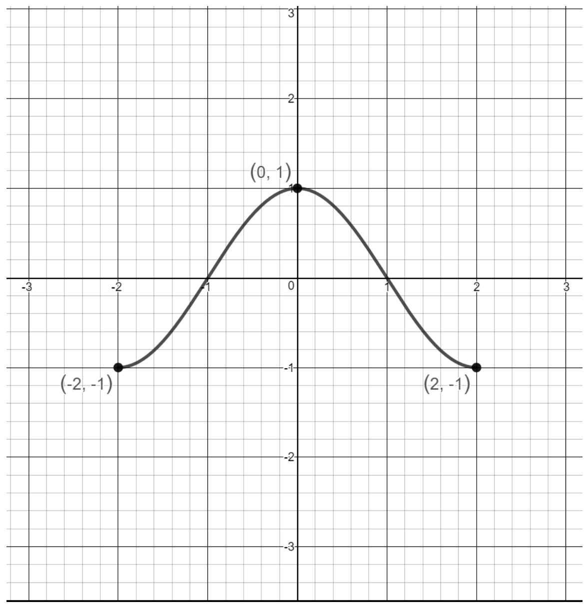
- $y = f(x)$ and $y = \frac{1}{a}f(x)$?

Divide all the y-values by a.

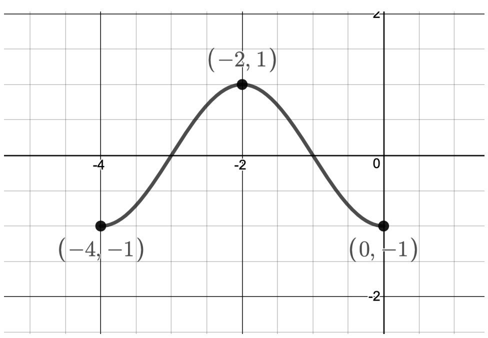
Compresses the graph vertically.

If $0 < a < 1$, multiplying by a compresses the graph vertically and dividing by a stretches the graph.

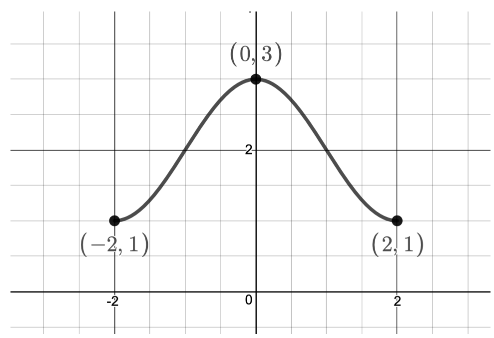
31. Below is the graph of a function $y = C(x)$. Use it to graph each of the functions below. How can you check your answers?



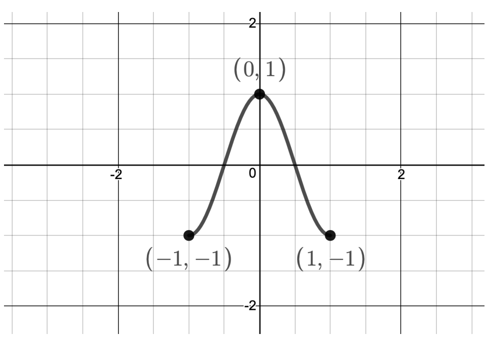
$y = C(x)$



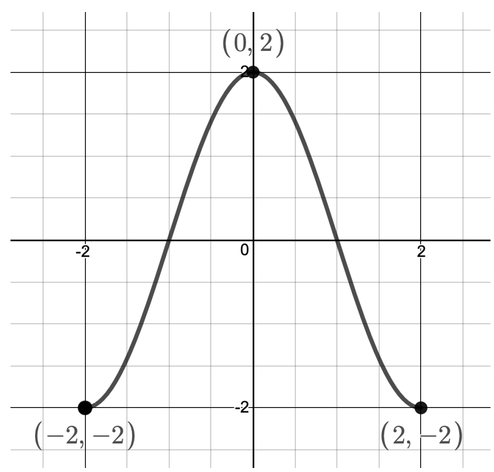
$y = C(x + 2)$



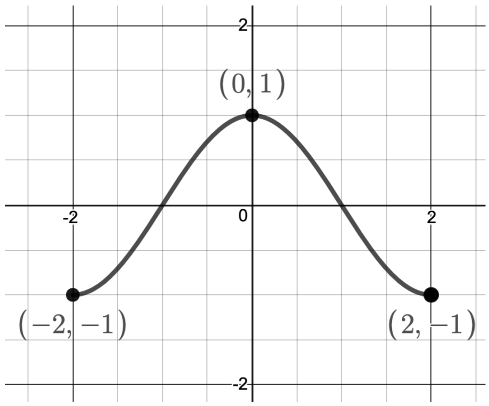
$y = C(x) + 2$



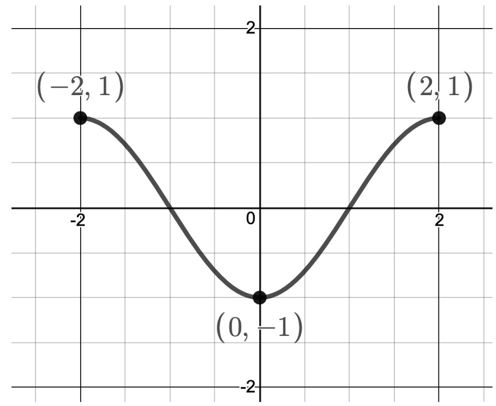
$y = C(2x)$



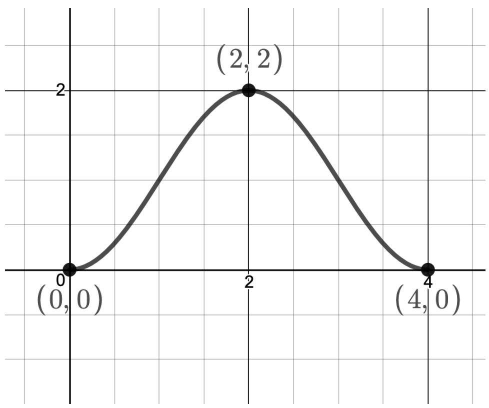
$y = 2C(x)$



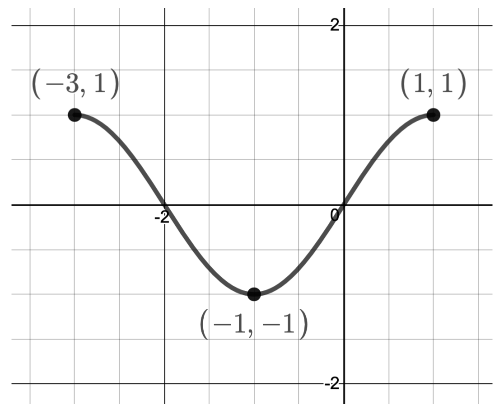
$y = C(-x)$



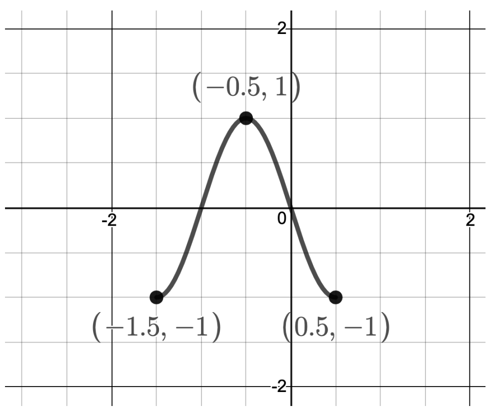
$y = -C(x)$



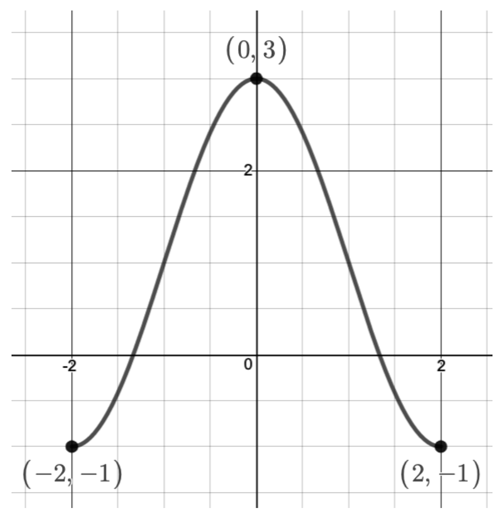
$y = C(x-2) + 1$



$y = -C(x+1)$



$y = C(2x+1)$



$y = 2C(x) + 1$